MAE 3240 Heat Transfer

Spring 2026

Homework Submission

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Problem set number:	#2	**Due date:**	02/10/2026

Acknowledgement:

No help

Problem 1

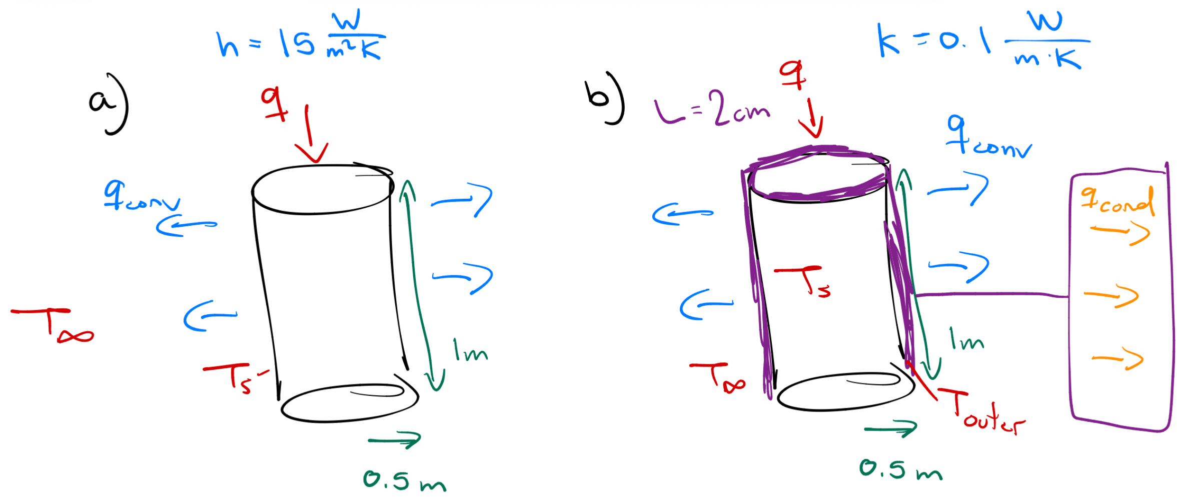
Known:

Cylinder with radius R = 0.5 m, H = 1 m. T_{water} = 50 °C. Exterior wall temperature T_s = 50 °C at steady state. T_∞ = 20 °C. For the water heater, h = 15 $\frac{W}{m^2 \cdot K}$. Thermal insulation k = 0.1 $\frac{W}{m \cdot K}$

Find:

a) Total electric heating power q to maintain T_{water} = 50 °C
b) The electrical heating power q needed to maintain T_{water} = 50°C. When an L = 2 cm thermal insulation form is attached to the exterior wall of the heater with thermal conductivity k = 0.1 $\frac{W}{m \cdot K}$
c) Compare the results from a) and b) and how thermal insulation affects the thermal insulation of the heater.

Schematic:



Assumptions:

Heater operated under steady state. Wall of water theater is thin enough to assume the exterior wall temperature is equal to the interior water temperature. The bottom of the water heated is well insulated (no heat transfer). Thermal radiation is negligible. For b), assume 1D heat conduction across the insulation foam.

Analysis:

a)
$q = hA(T_s - T_\infty)$ – heat transfer for convection
$A = \pi R^2 + 2\pi RH$ – add the sides plus the top
A = 3.927 m^2
Dimension check for q: $\frac{W}{m^2 \cdot K} \cdot m^2 \cdot K = W$
q = 1767.15 W
$q_{power} = |q|$
$q_{power} = 1767.15\,W$

b)
$q_{cond} = \frac{kA(T_s - T_{outer})}{L}$ – heat transfer for conduction
$q_{conv} = hA_{outer}(T_{outer} - T_\infty)$
At steady state, $q_{cond} = q_{conv}$ also, assume $A_{outer} = A$ (thin wall)

$$\frac{kA(T_s - T_{outer})}{L} = hA(T_{outer} - T\infty)$$

$T_{outer} = \frac{1}{A(h+k/L)} \cdot (kAT_s/L + hAT_\infty) = \frac{1}{h+k/L} \cdot (kT_s/L + hT_\infty)$

$T_\infty = 293.15\,K, T_s = 323.15\,K$
$T_{outer} = 300.65$ K
Dimensional check for q: $\frac{W}{m \cdot K} \cdot m^2 \cdot K \cdot \frac{1}{cm} \cdot \frac{100cm}{1m} = W$
$q = 441.79$ W
$q_{power} = |q|$
$q_{power} = 441.79\,W$

c)
With no insolation, the power required is 1767 W. With insolation 442 W is required. This means the insolation setup is about 4 times more efficient compared to the setup without insolation. So, when trying to design an efficient water heater, thermal insolation is very beneficial to minimizing the power required to work the water heater.

Problem 2

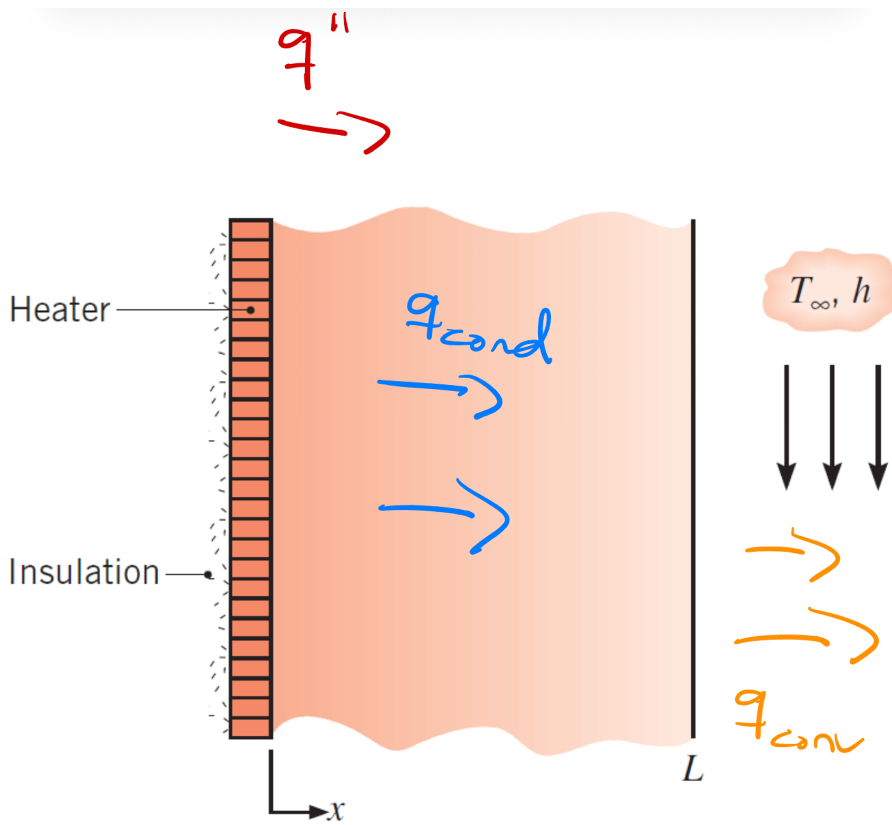
Known:

Thermally insulating wall with $L = 10\,\text{cm}$ and thermal conductivity of $k = 1\,\frac{W}{m\cdot K}$. Exterior surface of plastic layer has a coolant with temperature $T_\infty = 20°C$ and heat transfer coefficient $h = 100\,\frac{W}{m^2\cdot K}$.

Find:

a) If the heater provides $q'' = 1000\frac{W}{m^2}$ then find $T_{interior}(x = 0)$ and $T_{exterior}(x = L)$.
b) Given $T_{heater} \leq 200°C$, find q''_{max} from the heater.
c) What are 3 strategies that let the thin film heater be operated under higher heat flux without exceeding $200°C$?

Schematic:



Assumptions:

The system is at steady state. The heater is inifinitely thin. Thermal radiation is negligible.

Analysis:

a)
$T_\infty = 293.15\,K$ Continuity and steady state: $q'' = q''_{cond} = q''_{conv}$
$q''_{cond} = \frac{k}{L}(T_{interior} - T_{exterior})$
$q''_{conv} = h(T_{exterior} - T_\infty)$
There are 2 unknowns $(T_{exterior}, T_\infty)$ and 2 equations, so we can solve.
The only conversion we need to be worried about is L = 10 cm = 0.1 m
$T_{exterior} = \frac{q''}{h} + T_\infty$
$T_{exterior} = 303.15$ K
$T_{interior} = \frac{q''L}{k} + T_{exterior}$
$T_{interior} = 403.15K$

b)
Assume $T_{heater} = T_{interior}$ (thin wall assumption)
Use the same equations from a), but now q" is unknown and $T_{interior} = 200°C = 473.15\,K$.
$T_{exterior} = T_{interior} - \frac{q''L}{k}$
$q''_{max} = h(T_{exterior} - T_\infty)$

$$T_{exterior} = \frac{1}{1 + hL/k}(T_{interior} + \frac{T_\infty hL}{k})$$

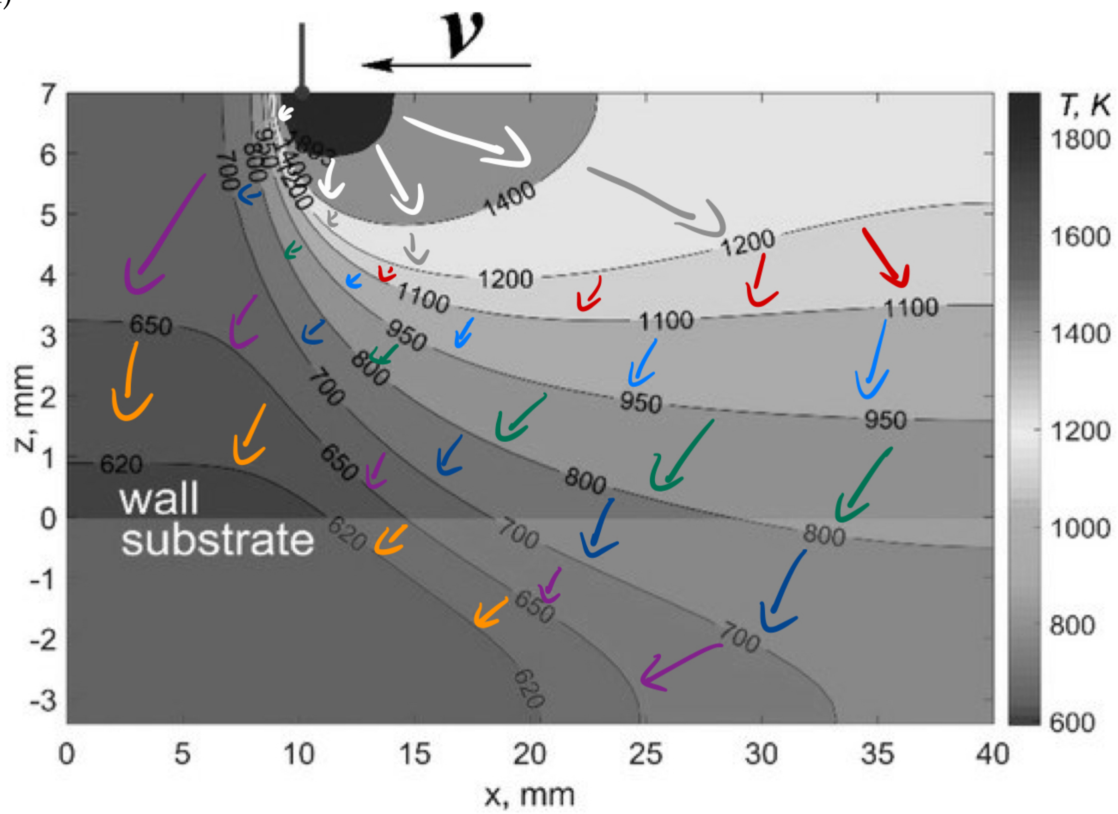
$T_{exterior} = 309.51$ K
$q''_{max} = 1636.0\frac{W}{m^2}$

c)
1) Increase the thermal conductivity (by changing the plastic layer) so that more heat gets transferred away from the heater meaning a higher heat capacity is possible while staying under 200°C.
2) Increasing heat transfer coefficient (and/or decreasing T_∞) by changing the coolant used would also help transfer heat away from the heater (where T_{max} occurs) and allow for a higher operating heat flux.
3) Decreasing L would also allow for more heat to be transferred by convection, which increases the maximum heat flux since convection is more efficient than conduction for heat transfer. However, if L is too low then the coolant will have more of an effect on the room temperature and the purpose of the heater may become moot.

Problem 3

Analysis:

a)



b)
The left side has the larger heat flux because the distance between the 1893 K and 1400 K contour line is significantly smaller compared to the right side. That means heat is able to transfer at a much faster rate on the left side. This requires that the heat flux of the left side must be larger.